

When equations (31) through (34) are combined with (10), the effective strain becomes

$$\bar{\epsilon} = (\alpha_1 r^4 + \beta_1 r^2 + \gamma_1)^{1/2} \quad (35)$$

where

$$\alpha_1 = \frac{52}{3} a_1^2, \quad \gamma_1 = 4(3a_2 z^2 + a_3)^2 \quad (36)$$

$$\beta_1 = \frac{4}{3} [(12a_1 a_2 + 9a_2^2 + 16a_1^2) z^2 + 12a_1 a_3]$$

The first derivative with respect to z of the above coefficients will be required later, and are documented here as

$$\beta_1' = \frac{8}{3} (12a_1 a_2 + 9a_2^2 + 16a_1^2) z \quad (37)$$

$$\gamma_1' = 48a_2 (3a_2 z^2 + a_3) z$$