When equations (31) through (34) are combined with (10), the effective strain becomes

$$
\begin{equation*}
\bar{\epsilon}=\left(\alpha_{1} r^{4}+\beta_{1} r^{2}+\gamma_{1}\right)^{1 / 2} \tag{35}
\end{equation*}
$$

where

$$
\begin{aligned}
& \alpha_{1}=\frac{52}{3} a_{1}^{2}, \quad r_{1}=4\left(3 a_{2} z^{2}+a_{3}\right)^{2} \\
& \beta_{1}=\frac{4}{3}\left[\left(12 a_{1} a_{2}+9 a_{2}^{2}+16 a_{1}^{2}\right) z^{2}+12 a_{1} a_{3}\right]
\end{aligned}
$$

The first derivative with respect to $z$ of the above coefficierts will be required later, and are documented here as

$$
\begin{align*}
& \beta_{1}^{\prime}=\frac{8}{3}\left(12 a_{1} a_{2}+9 a_{2}^{2}+16 a_{1}^{2}\right) z  \tag{37}\\
& r_{1}^{\prime}=48 a_{2}\left(3 a_{2} z^{2}+a_{3}\right) z
\end{align*}
$$

